On the WKB Quantum Equivalence between Diverse p–brane Actions

Stefano Ansoldi*

Dipartimento di Fisica Teorica, Università di Trieste,

INFN, Sezione di Trieste

C.Castro†

Center for Theoretical Studies of Physical Systems

Clark Atlanta University, Atlanta, GA.30314

Euro Spallucci‡

Dipartimento di Fisica Teorica, Università di Trieste,

INFN, Sezione di Trieste

Abstract

We consider an action for a closed, bosonic, p–brane, where the brane tension is not an assigned parameter but rather it is induced by a maximal rank gauge p–form. This model is classically equivalent to the Nambu–Goto/Howe–Tucker model. We investigate how this classical equivalence can be implemented in the path integral framework. For this purpose we adopt a "first order" integration procedure over gauge p–forms and a "shortened" Fadeev–Popov procedure.

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*E-mail address: ansoldi@trieste.infn.it

†E-mail address: castro@ctsps.cau.edu

‡E-mail address: spallucci@trieste.infn.it

1

Diverse action functionals have been proposed to describe dynamics of a relativistic, bosonic, p–brane [1]. The first brane action, proposed in 1975, was a generalization of the Nambu–Goto action for strings, i.e. the measure of the brane world history [2]

$$S_{DNG}[Y] = -m_{p+1} \int_\Sigma d^{p+1}\sigma\sqrt{-\gamma}\,, \quad \gamma \equiv det\,(\partial_m Y^\mu \partial_n Y_\mu) \quad, \tag{1}$$

where m_{p+1} is the "p–tension". We use "p" for the spatial dimensionality of the brane; thus, the coordinates σ^m, $m = 0\,,1\,,\dots p$, span the $(p+1)$–dimensional world manifold Σ. The D functions $Y^\mu(\sigma)$, $\mu = 0\,,1\,,\dots D$, are the brane coordinates in the D–dimensional target spacetime. The special case $p = 2$ and $D = 4$ was already introduced in 1962 by Dirac in an attempt to resolve the electron–muon puzzle in terms of a relativistic membrane [3].

An alternative description, preserving world manifold reparametrization invariance, can be achieved by introducing an auxiliary world manifold metric $g_{mn}(\sigma)$ and a "cosmological term" [4], [5],

$$S_{HTP}[Y\,,g] = -\frac{m_{p+1}}{2}\int_\Sigma d^{p+1}\sigma\sqrt{-g}\,[\,g^{mn}\partial_m Y^\mu\,\partial_n Y_\mu - (p-1)\,] \quad, \tag{2}$$

where $g \equiv \det(g_{mn})$. In both functionals (1) and (2) the brane tension m_{p+1} is a *pre–assigned* parameter.

The two actions (1) and (2) are classically equivalent as the "field equations" $\delta S/\delta g^{mn}(\sigma) = 0$ require the auxiliary world metric to match the induced metric, i.e. $g_{mn} = \gamma_{mn} = \partial_m Y^\mu \partial_n Y_\mu$. Moreover they are also complementary: S_{DNG} provides an "extrinsic" geometrical description in terms of the embedding functions $Y^\mu(\sigma)$ and the induced metric γ_{mn}, while S_{HTP} assigns an "intrinsic" geometry to the world manifold Σ in terms of the metric g_{mn} and the "cosmological constant" m_{p+1}; the $Y^\mu(\sigma)$ functions enter as a "multiplet of scalar fields" propagating on a curved $(p+1)$–dimensional manifold.

More recently new action functionals have been proposed where the brane tension, or world manifold cosmological constant, is not an *a priori* assigned parameter, but follows from the dynamics of the object itself and can attain both positive and vanishing values. Either Kaluza–Klein type mechanism [6] and modified integration measure [7] have been proposed as candidate dynamical processes to produce tension at the classical level.

The main purpose of this note is to investigate how the dynamical generation of the brane tension and the equivalence between diverse action functionals can be extended at the quantum level in the WKB approximation of a "sum over histories" approach. However, before considering the path–integral it is instrumental to review how classical dynamics leads to the action (1) as an *effective,* on–shell action.

The guiding principle to assign the p–brane tension the role of a dynamical variable is borrowed from modern cosmology, where the cosmological constant can be represented by a *maximal rank* gauge p–form [8]. Thus, we introduce the following action functional

$$S[\,Y\,,g\,,A\,] = -\int_\Sigma d^{p+1}\sigma \sqrt{-g}\left[\frac{m_{p+1}}{2}\frac{(-\gamma)}{(-g)} - \frac{1}{2(p+1)!}F_{m_1\ldots m_{p+1}}F^{m_1\ldots m_{p+1}}\right]$$

$$= -\int_\Sigma d^{p+1}\sigma \left[\frac{m_{p+1}}{2}\frac{(-\gamma)}{\sqrt{-g}} - \frac{\sqrt{-g}}{2(p+1)!}F_{m_1\ldots m_{p+1}}F^{m_1\ldots m_{p+1}}\right] \quad, \qquad (3)$$

where the world manifold Σ has a space–like boundary $\partial\Sigma$ whose target space image will represent a closed, p–dimensional, relativistic object. Moreover, we introduce on the world manifold a maximal rank gauge field, $A_{m_2\ldots m_{p+1}}(\sigma)$, with field strength $F_{m_1\ldots m_{p+1}} \equiv \partial_{[\,m_1}A_{m_2\ldots m_{p+1}]}(\sigma)$. To preserve gauge invariance under $\delta A_{m_2\ldots m_{p+1}} = \partial_{[\,m_2}\Lambda_{m_3\ldots m_{p+1}]}$ in the presence of a boundary we must give up a current–potential interaction term and consider *only* a gravitational coupling A–g. By a suitable rescaling of the brane coordinates the dimensional constant m_{p+1} can be washed out, and the classical action (3) written without any dimensional scale. Our goal is to show that the Dirac–Nambu–Goto functional can be obtained as an *effective* action from (3) once the classical field equations for the p–form gauge potential are solved.

Varying the action (3) with respect to A we get

$$\frac{\delta S[\,Y\,,g\,,A\,]}{\delta A_{m_2\ldots m_{p+1}}(\sigma)} = 0 \quad\longrightarrow\quad \partial_m\left(\sqrt{-g}\,F^{mm_2\ldots m_{p+1}}\right) = 0 \quad,$$

which, since A is maximal on the p–brane, has the solution

$$F^{mm_2\ldots m_{p+1}} = \Lambda\epsilon^{mm_2\ldots m_{p+1}} = \Lambda\frac{1}{\sqrt{-g}}\delta^{[\,mm_2\ldots m_{p+1}]} \quad, \qquad (4)$$

where Λ is an arbitrary integration constant. By inserting the solution (4) back into (3) we obtain

$$S[\,Y\,,g\,] = -\int_\Sigma d^{p+1}\sigma \left[\frac{m_{p+1}}{2}\frac{(-\gamma)}{\sqrt{-g}} + \frac{\Lambda^2}{2}\sqrt{-g}\right] \quad, \tag{5}$$

where, the world manifold cosmological constant Λ^2 shows up as the *on–shell* value of the gauge field kinetic term.

The on-shell action (5) depends from the world metric *only* through the volume density $\sqrt{-g}$. Hence, variations with respect to g_{mn} reduce to variations with respect $\sqrt{-g}$:

$$\frac{\delta S[\,Y\,,g\,]}{\delta g_{mn}(\sigma)} = 0 \quad\longleftrightarrow\quad \frac{\delta S[\,Y\,,g\,]}{\delta\sqrt{-g}} = 0 \quad. \tag{6}$$

By inserting the solution (4) into (6) we get

$$m_{p+1}\frac{(-\gamma)}{(-g)} = \Lambda^2 \quad\Rightarrow\quad \sqrt{-g} = \frac{1}{\Lambda}\sqrt{m_{p+1}}\sqrt{-\gamma} \tag{7}$$

and

$$S = -\Lambda\sqrt{m_{p+1}}\int_\Sigma d^{p+1}\sigma\,\sqrt{-\gamma} \equiv -\rho_p\int_\Sigma d^{p+1}\sigma\,\sqrt{-\gamma} \quad. \tag{8}$$

After solving for the world metric in terms of the brane coordinates, the action (3) turns out to be equivalent to a Dirac–Nambu–Goto action with a dynamically induced brane tension given by $\rho_p \equiv \Lambda\sqrt{m_{p+1}}$. Let us remark that Λ can take any value including zero. Accordingly, *null branes*, corresponding to the action

$$S_{null}[\,Y\,,g\,] = -\frac{m_{p+1}}{2}\int_\Sigma d^{p+1}\sigma\,\frac{(-\gamma)}{\sqrt{-g}} \quad, \tag{9}$$

are included in our description as well. This special case stresses how the parameter m_{p+1} is not necessarily the brane tension, but only a dimensional constant needed to allow the various dynamical fields in the action to keep their canonical dimensions[1].

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[1] If we keep $m_{p+1} \neq 0$ the brane coordinates have canonical dimension of length, while the world and the induced metric are dimensionless (in units $\hbar = 1$, $c = 1$), i.e. $[\,Y^\mu\,] = M^{-1}$, $[\,\gamma_{mn}\,] = [\,g_{mn}\,] = 1$.

In the second part of this note we shall discuss the above equivalence at the quantum level. The basic quantity encoding the p–brane quantum dynamics is the *boundary wave functional,* or vacuum—one–brane amplitude

$$Z \equiv Z[\widehat{Y} , \widehat{A} , \widehat{g}] = \int^{\widehat{g}} [Dg_{mn}] \int^{\widehat{Y}} [DY^{\mu}] \int^{\widehat{A}} [DA] \exp(iS[Y , g , A]) \quad , \qquad (10)$$

where the sum is over all bulk fields configurations inducing "hatted" fields on the boundary of the brane. We are assuming that the brane world manifold has a single, p–dimensional boundary, parametrized as $\sigma^m = \sigma^m(s^a)$, $a = 1 , \dots p$, which is mapped into the physical brane $\widehat{Y}^{\mu}(s)$; $\widehat{g}$ and $\widehat{A}$ are the induced metric and gauge potential over $\widehat{Y}^{\mu}(s)$. The integration variables in Z "live" in the brane bulk, while we let free the fields induced on the boundary, i.e. we do not assign an independent classical action to the hatted fields.

The first field to be integrated out is the gauge p–form A. The standard routine goes through a lengthy procedure of gauge fixing and Fadeev–Popov compensation to invert the classical kinetic operator and define an appropriate quantum propagator. On the other hand, one knows that a gauge p–form over a $(p+1)$–dimensional manifold has no dynamical degrees of freedom and can describe only a static interaction. In such a limiting case the Fadeev–Popov procedure leaves no propagating degree of freedom at the quantum level. To shorten the whole gauge fixing procedure of ghost terms with different rank [9], we shall provide an alternative "recipe" to kill all the apparent degrees of freedom. We write the path–integral in the first order version, where the gauge potential A and field strength F are introduced as independent integration variables [10] and we integrate away the gauge part of A after inserting gauge fixing Dirac delta's and the corresponding ghost determinants in the functional measure. The remaining, gauge invariant part of A enforces F to be a classical solution of the field equations, which is a constant background field. No propagating degrees of freedom survive at the quantum level. A formal proof of the equivalence between second order and first order quantization procedures, in the general case of a p–form in $p + 1$ dimensions, is beyond the purpose of this short note. Rather, we will briefly consider the simplest, non trivial case which is $p = 1$ gauge form over a two-dimensional, flat manifold

without boundary, and then translate the result to the case we are studying. The first order, gauge fixed and Fadeev–Popov compensated path integral is

$$Z_{p=1} = \int [\, DF \,][\, DA \,] \delta \,[\, \partial^m A_m \,] \Delta_{FP} \exp \left\{ i \int d^2\sigma \left[\frac{1}{4} F_{mn} F^{mn} - \frac{1}{2} F^{mn} \partial_{[m} A_{n]} \right] \right\} \quad . \quad (11)$$

By splitting A_m into the sum of a "transverse" vector A_m^T and a "gauge part" $\partial_m \phi$, the integration measure $[\, DA \,]$ turns into $[\, DA^T \,][\, D\phi \,] (\det \Box)^{1/2}$ and (11) reads

$$Z_{p=1} = \int [\, DF \,][\, DA^T \,][\, D\phi \,] (\det \Box)^{1/2} \delta \,[\, \Box \phi \,] \times$$

$$\times \Delta_{FP} \exp \left\{ i \int d^2\sigma \left[\frac{1}{4} F_{mn} F^{mn} - \frac{1}{2} F^{mn} \partial_{[m} A_{n]}^T \right] \right\}$$

$$= \int [\, DF \,][\, DA^T \,] (\det \Box)^{1/2} \exp \left\{ i \int d^2\sigma \left[\frac{1}{4} F_{mn} F^{mn} + \frac{1}{2} A_n^T \partial_m F^{mn} \right] \right\} \quad , \quad (12)$$

where the the gauge part has been integrated away thanks to the Fadeev–Popov determinant $\Delta_{FP} = \det \Box$ and only the gauge invariant A^T vector remains in the classical action. The extra Jacobian, coming from the change of the integration measure, will be cancelled in a while when integrating F. The pay off for relaxing the relationship between A and F and getting rid of the gauge part is that A^T *linearly* enters the first order action, i.e. A^T plays the role of Lagrange multiplier imposing F to satisfy the classical field equations

$$Z_{p=1} = \int [\, DF \,] (\det \Box)^{1/2} \delta \,[\, \nabla_m F^{mn} \,] \exp \left\{ i \int d^2\sigma \left[\frac{1}{4} F_{mn} F^{mn} \right] \right\}$$

$$= \int [\, DF \,] \delta \,[\, F^{mn} - \Lambda \epsilon^{mn} \,] \exp \left\{ i \int d^2\sigma \left[\frac{1}{4} F_{mn} F^{mn} \right] \right\} \quad . \quad (13)$$

Equation (13) shows that the first order formulation of a limiting rank, abelian gauge theory, and the Fadeev–Popov prescription lead to a *"trivial"* path integral for F. The Dirac–delta picks up the classical configurations of the world tensor F and the whole path–integral "collapses" around the *classical trajectory*[2]. Thus, F is "frozen" to a constant value Λ and

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[2]The same kind of "collapse" around the classical trajectory has been introduced in string theory [11] to pick up the Eguchi "Area dynamics" [12]. For a pedagogical introduction to this new path–integral manipulation see [13], where it has been applied to a simpler case, the non–relativistic point particle propagator.

no degrees of freedom are left free to propagate. The same result can be obtained, with some additional work, for $p > 1$ as well. Accordingly, we get

$$
\begin{aligned}
Z &= \exp\left\{ -\frac{1}{p!} \int_{\partial\Sigma} dN_{k_1} \sqrt{-\widehat{g}}\, \widehat{F}^{k_1\ldots k_{p+1}}\, \widehat{A}_{k_2\ldots k_{p+1}} \right\} \times \\
&\quad \times \int [\,DF\,]\,(\det \square)^{1/2}\, \delta\left[\partial_{m_1}\left(\sqrt{-g}F^{m_1 m_2\ldots m_{p+1}} \right) \right] \times \\
&\quad \times \exp\left(\frac{i}{2(p+1)!} \int_\Sigma d^{p+1}\sigma \sqrt{-g}\, F^2_{m_1\ldots m_{p+1}} \right) \\
&= \exp\left\{ -\frac{i\Lambda}{p!} \int_{\partial\Sigma} \widehat{A}_{k_1\ldots k_{p+1}}\, ds^{k_1} \wedge \ldots \wedge ds^{k_p} \right\} \exp\left(-\frac{i\Lambda^2}{2} \int_\Sigma d^{p+1}\sigma \sqrt{-g} \right) \quad .
\end{aligned}
\tag{14}
$$

The first term in (14) is a pure boundary quantity produced by a partial integration of the term $F\partial_{[.}A_{.]}$. A similar term arises in string theory when boundary and bulk quantum dynamics are properly split [11].

After integrating out the gauge degrees of freedom the resulting path–integral reads

$$
\begin{aligned}
Z &= \int^{\widehat{g}} [\,Dg_{mn}\,] \int^{\widehat{Y}} [\,DY^\mu\,] \exp\left(-\frac{i\Lambda}{p!} \int_{\partial\Sigma} \widehat{A}_{k_1\ldots k_{p+1}}\, ds^{k_1} \wedge \ldots \wedge ds^{k_p} \right) \times \\
&\quad \times \exp\left(-i \int_\Sigma d^{p+1}\sigma \left[\frac{m_{p+1}}{2} \frac{(-\gamma)}{\sqrt{-g}} + \frac{\Lambda^2}{2}\sqrt{-g} \right] \right) \\
&\equiv \int^{\widehat{g}} [\,Dg_{mn}\,] \int^{\widehat{Y}} [\,DY^\mu\,] W_{\widehat{A}}[\,\partial\Sigma\,] \exp\left(-i \int_\Sigma d^{p+1}\sigma \left[\frac{m_{p+1}}{2} \frac{(-\gamma)}{\sqrt{-g}} + \frac{\Lambda^2}{2}\sqrt{-g} \right] \right) \quad .
\end{aligned}
\tag{15}
$$

We remark that this integration procedure is *exact* and leads to a bulk action plus a boundary correction represented by the generalized Wilson factor $W_{\widehat{A}}[\,\partial\Sigma\,]$.

We also notice that the world metric enters the path–integral only through the world volume density. Accordingly, we can "change" integration variable

$$
\int^{\widehat{g}} [\,Dg_{mn}\,] \quad \longrightarrow \quad \int^{\widehat{g}} [\,Dg_{mn}\,] \int^{\widehat{e}} [\,De\,]\delta\left[e(\sigma) - \sqrt{-g} \right]
\tag{16}
$$

and write (15) as

$$
Z = \int^{\widehat{e}} [\,De\,] \int^{\widehat{Y}} [\,DY^\mu\,] W_{\widehat{A}}[\,\partial\Sigma\,] \exp\left(-i \int_\Sigma d^{p+1}\sigma \left[\frac{m_{p+1}}{2} \frac{(-\gamma)}{e(\sigma)} + \frac{\Lambda^2}{2} e(\sigma) \right] \right) \quad .
\tag{17}
$$

The saddle point value for the auxiliary field $e(\sigma)$ is defined by:

$$
\frac{\delta S}{\delta e(\sigma)} = 0 \quad \longrightarrow \quad e_{cl.}(\sigma) = \frac{1}{\Lambda} \sqrt{m_{p+1}} \sqrt{-\gamma} \quad .
\tag{18}
$$

By expanding Z around the saddle point $e_{cl.}(\sigma)$ we obtain the Dirac–Nambu–Goto path–integral. Correspondingly, we get the following *semi–classical* equivalence relation

$$Z = \int^{\widehat{g}} [\, Dg_{mn}\,] \int^{\widehat{Y}} [\, DY^{\mu}\,] \int^{\widehat{A}} [\, DA\,] \times$$

$$\times \exp\left[-im_{p+1} \int_{\Sigma} d^{p+1}\sigma \frac{(-\gamma)}{2\sqrt{-g}} - \frac{i}{2(p+1)!} \int_{\Sigma} d^{p+1}\sigma \sqrt{-g}\, F^2_{m_1\dots m_{p+1}}(A) \right]$$

$$\approx \int [\, DY^{\mu}\,] W_{\widehat{A}}[\,\partial\Sigma\,] \exp\left[-i\rho_p \int_{\Sigma} d^{p+1}\sigma \sqrt{-\gamma} \right] \quad . \tag{19}$$

The extension of the relation (19) beyond the saddle point approximation is currently under investigation, and requires a proper treatment of the p–brane degrees of freedom at the quantum level. As is well known, bosonic branes viewed as non–linear σ–models are non–renormalizable perturbative quantum field theories when $p > 1$. However, we can look at p–branes not as σ–models but as elements of a more fundamental theory, say M-Theory, which is in essence a non–perturbative theory. This approach is not new. For example the Einstein–Hilbert action in three and four dimensions is not perturbatively renormalizable; neverthless, three dimensional Einstein–Hilbert gravity can be reformulated as a Chern–Simon gauge theory which can be be exactly solved at the quantum level [14]. In a similar way, four dimensional General Relativity can be written in terms of Ashtekar variables which provides an exact formulation of non-perturbative canonical quantum gravity [15]. From the same point of view, we think that perturbation theory is not the ultimate way to approach the problem of brane quantization. Moreover, a supersymmetric membrane in $D = 11$ spacetime dimensions is expected to be a finite quantum model [16], where both ultraviolet and infrared divergences are kept under control. For a general discussion of quantum super-membranes we refer to [17], and limit our considerations to the semi-classical level. Hopefully, a proper understanding of M-Theory will provide a background independent formulation of string/brane theory where the quantum path–integral will be well defined. In the meanwhile, we shall work in the WKB approximation where one can choose the action (3), in place of (1) or (2), as a starting point. The non–linearity and reparametrization invariance of the Nambu–Goto action make difficult, if not impossible, to implement the original Feynman construction of the path–integral as a sum of *phase space trajectories* [18]. One

is forced, almost unavoidably, to resort to standard perturbative approaches, e.g. normal modes expansion, or sigma–model effective field theory. Any perturbative approach captures some dynamical feature and misses all the other ones. The impressive results obtained in string theory through duality relations of several kind [19] show that what is not accessible in a given perturbation scheme can be obtained through a different one. With this in mind, we hope that an action of the type (3), where the brane variables enter polynomially, and the tension is brought in by a generalized gauge principle, can be more appropriate to implement the Feynman's original proposal, or, at least, to provide a *different "perturbative" quantization scheme for the Nambu–Goto model itself.* In such a, would be, "new regime" of the Nambu–Goto brane both massive and massless objects are present at once and correspond to different values of the world manifold strength $F_{m_1...m_{p+1}}$. It would be tempting to assign F the role of "order parameter" and describe the dynamical generation of the brane tension as a sort of phase transition. Furthermore, it would be interesting to extend the action (3) in order to include negative tension branes as well. This kind of objects appear to play an important role in the realization of the brane world scenario [20], [21].

All these problems, are currently under investigation and eventual results will be reported in future publications.

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